Teaching materials

Guide notes 2. Velocity analysis of a four-bar mechanism

MISCE project

Mechatronics for Improving and Standardizing Competences in Engineering

Competence: Mechanical Engineering

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This document corresponds to the second lecture for the competence ‘Mechanical Engineering’ using the ‘Four-bar mechanism platform’

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1 Objective

The main objective of this lesson is to deepen students' understanding of planar kinematics through the velocity analysis of a four-bar mechanism.

The session focuses on calculating angular velocities and point velocities within the mechanism based on a known input motion. Students will solve the velocity problem using analytical and numerical methods and validate their results through experimental measurements obtained from the four-bar mechanism platform.

2 Context

The mechanical system employed is a four-bar mechanism driven by a DC-motor. The input signal to this motor is a pulse width modulated (PWM) signal sent and controlled via a MATLAB-based control app (see Lesson 0. Introduction to the experimental platform).

This experiment enables students to calculate the angular velocity of each link and the velocity of points of interest based on a known input speed. The lesson builds on the position analysis and deepens the understanding of motion transmission within closed-loop systems. In general, students gain practical insight into the dynamic behaviour of mechanisms and learn to correlate theoretical predictions with experimental data.

3 Velocity analysis of a four-bar mechanism

The velocity analysis consists of calculating the angular velocities of all other links and the velocities of points of interest, given the known positions of all the links in the mechanism and the angular velocity of the input link. All these velocities are referenced to the fixed link, link 2, and are therefore absolute velocities, which are required for working with complex numbers.

To perform this analysis, we differentiate the position equation and obtain:

$$\dot{\boldsymbol{r}}_2 + \dot{\boldsymbol{r}}_3 = \dot{\boldsymbol{r}}_1 + \dot{\boldsymbol{r}}_4$$

To solve the velocity problem, we can either differentiate the loop equation directly or substitute each $\boldsymbol{r}_i$ with its expression from equation $\dot{\boldsymbol{r}} = \dot{\rho} e^{i\theta} + \rho\omega i e^{i\theta}$, omitting terms as appropriate. In this case, since the complex vectors represent the positions between rotational pairs and the distances between them do not change, all terms involving $\dot{\rho}_i$ are zero. Also, $\dot{\theta}_1$ is zero because the fixed link does not rotate. This leads to:

$$\cancel{\dot{\rho}_2} e^{i\theta_2} + \rho_2\dot{\theta}_2 i e^{i\theta_2} + \cancel{\dot{\rho}_3} e^{i\theta_3} + \rho_3\dot{\theta}_3 i e^{i\theta_3} = \cancel{\dot{\rho}_1} e^{i\theta_1} + \rho_1\cancel{\dot{\theta}_1} i e^{i\theta_1} + \cancel{\dot{\rho}_4} e^{i\theta_4} + \rho_4\dot{\theta}_4 i e^{i\theta_4}$$

Renaming $\dot{\theta}_i$ as ω_i, the equation becomes

$$\omega_2\rho_2 i e^{i\theta_2} + \omega_3\rho_3 i e^{i\theta_3} = \omega_4\rho_4 i e^{i\theta_4}$$

and using equation $\boldsymbol{r} = \rho e^{i\theta}$, we can split the previous equation into two scalar equations corresponding to the real and imaginary parts:

$$-\omega_2\rho_2 sin(\theta_2) - \omega_3\rho_3 \sin(\theta_3) = -\omega_4\rho_4 \sin(\theta_4)$$
$$\omega_2\rho_2 cos(\theta_2) + \omega_3\rho_3 \cos(\theta_3) = \omega_4\rho_4 \cos(\theta_4)$$

which can be written in matrix form as:

$$\begin{bmatrix} -\rho_3\sin(\theta_3) & \rho_4\sin(\theta_4) \\ \rho_3\cos(\theta_3) & -\rho_4\cos(\theta_4) \end{bmatrix} \begin{Bmatrix} \omega_3 \\ \omega_4 \end{Bmatrix} = \begin{Bmatrix} \omega_2\rho_2\sin(\theta_2) \\ -\omega_2\rho_2\cos(\theta_2) \end{Bmatrix}$$

This system of equations, with ω_3 and ω_4 as unknowns, is linear and can be easily solved using Cramer's rule. Note that the lengths of the links are defined by the student.

On the other hand, the angles of the links are either given (like θ_2) or have been calculated in the position analysis.

4 Methodology

4.1 Example: Solving the velocity problem

For the example in this document, the given geometrical values for the four-bar mechanism's links are:

- $\rho_1 = 0.14$ m
- $\rho_2 = 0.05$ m
- $\rho_3 = 0.16$ m
- $\rho_4 = 0.1$ m

Using the equations presented in the previous section and based on the angles from Table 1, it is easy to determine the resulting angular velocities ω_4, when considering $\omega_2 = 1$ rad/s constant:

Table 1. Angular velocity ω_4 for $\omega_2 = 1$ rad/s

θ_2 (°)	ω_4 (rad/s)
45	0.3159
135	0.4644
225	0.0013
315	-0.7175

The evolution of ω_4 will follow a typical cyclic pattern when the input velocity is constant. However, with a uniformly accelerated input velocity the resulting graph will no longer have equal periods. Figure 1 shows these analytical graphs of ω_4 over time for the example used in this lab exercise.

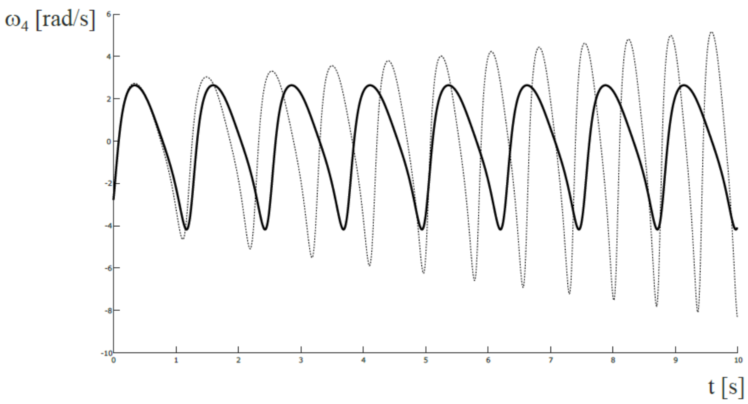

Figure 1. Angular velocity ω_4 over time for constant input velocity (bold line) and uniformly accelerated input ω_2 (thin dashed line)

4.2 Student task

The student must rotate the motor driving link, the crank or link 2, and record the angular velocity of link 4, the rocker, using the optical encoder. They will then analytically solve for the output angular velocity ω_4, as explained in the theoretical lessons. The goal of this exercise is to verify that the analytically calculated velocities match the experimental results.

Additionally, the student will repeat the experiment with a uniformly accelerated input velocity. In this case, the resulting graph will no longer have equal periods, i.e. the angular velocity increases over time, and the periods become shorter.